1

P1.1.2 A lamp assembly is connected to a battery and a switch by a wire 2 m long and 1 mm^2 cross section. If a current of 1 A flows in the wire, determine the time it takes a conduction electron to travel along the 2 m length of wire from the switch to the lamp assembly, assuming the concentration of conduction electrons in the wire is 8.4×10^{28} /m^3 and the charge on an electron is 1.6×10^{-19} C (Refer to Equation 1.4.3). Note that the delay in the turning on of the lamp assembly after the switch is closed does not depend on the time it takes a conduction electron to travel from the switch to the lamp assembly, since these electrons are present all along the wire at any given time. The delay depends on how fast the electric field propagates, after the switch is closed, so as to act on the conduction electrons at the end of the wire in contact with the lamp assembly. The speed of propagation of the electric filed along the wire is of the order of the speed of light.

Solution: $|u| = \dfrac{1\text{A}}{10^{-6}\times8.4\times10^{28}\times1.6\times10^{-19}} = 7.44\times10^{-5}$ m/s; $t = \dfrac{2\,\text{m}}{7.44\times10^{-5}} \cong 26880$ s $\equiv$ 7 hrs, 28 min.

P1.1.7 The charge q varies with time as shown in Figure P1.1.7, where $q = 2\sin(\pi t/2)$ C, $0 \le t \le 1$ s. Determine the variation of the current i with time.

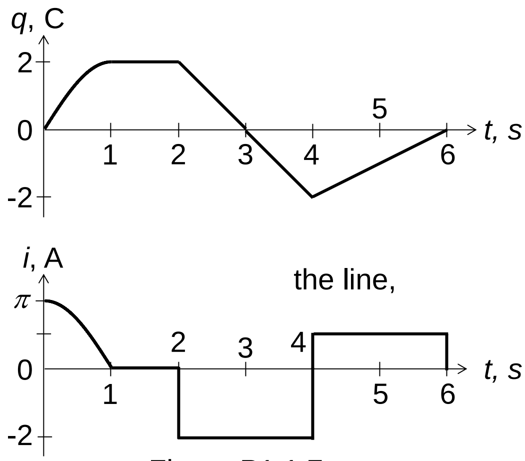

Figure P1.1.7

Solution: $i = \dfrac{dq}{dt} = \pi\cos(\pi t/2)$ A, $0 \le t \le 1$ s; $i = 0$, $1 \le t \le 2$ s; for $2 \le t \le 4$ s; i is the slope of the line, which is -2 C/s = -2 A; similarly, for $4 \le t \le 6$ s, the slope is 2 C/2s = 1 A. The time variation is as shown. Note that q starts and ends at zero, so that the total area under the i-t should be zero. This area is $\int_0^1 \pi\cos(\pi t/2)dt$ – 4 + 2 = $\left[2\sin(\pi t/2)\right]_0^1 = 2 - 4 + 2 = 0$.

P1.1.8 Consider that Figure P1.1.8 represents the variation of the current i in A with time. Determine the variation of q as a function of t during each interval and verify the values at the end of each interval by calculating the area involved.

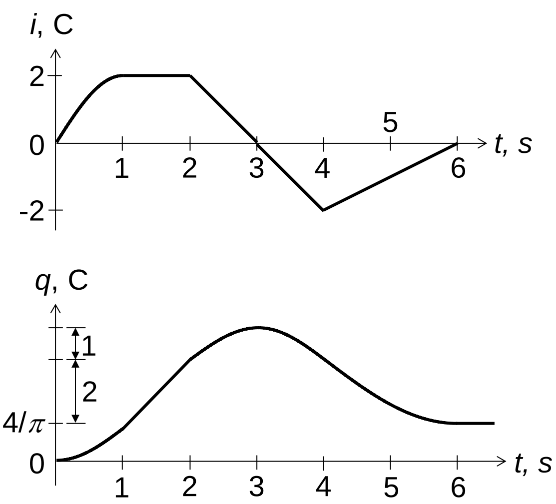

Figure P1.1.8

Solution: <u>$0 \le t \le 1$ s:</u> $i = 2\sin(\pi t/2)$ A,

$$q = \int_0^t 2\sin\left(\frac{\pi}{2}t\right)dt = \frac{4}{\pi}\left[-\cos\left(\frac{\pi}{2}\right)t\right]_0^t$$

$$q = \frac{4}{\pi}\left(1 - \cos\left(\frac{\pi t}{2}\right)\right) = \frac{4}{\pi} \text{ at } t = 1 \text{ s.}$$

<u>$1 \le t \le 2$ s:</u> $i = 2$ A; $q = \int_1^t 2dt + q\big|_{t=1}$, $q = 2t - 2 + \dfrac{4}{\pi} = 2 + \dfrac{4}{\pi}$ at $t = 2$ s. The area added is that of the rectangle, which is 2 C.

<u>$2 \le t \le 4$ s:</u> $i = 2(3 - t)$ A; $q = \int_2^t 2(3 - t)dt + q\big|_{t=2} = \left[6t - t^2\right]_2^t + 2 + \dfrac{4}{\pi} =$

$-t^2 + 6t - 6 + \dfrac{4}{\pi} = 3 + \dfrac{4}{\pi}$ at $t = 3$ s, and $2 + \dfrac{4}{\pi}$ at $t = 4$ s. At $t = 3$ s, the area that is added to that at $t = 2$ s is 1, the area of the triangle, and this same area is subtracted at $t = 4$.

<u>$4 \le t \le 6$ s:</u> $i = t - 6$ A; $q = \int_4^t (t - 6)dt + q\big|_{t=4} = \left[\dfrac{t^2}{2} - 6t\right]_4^t + 2 + \dfrac{4}{\pi} =$

$\dfrac{t^2}{2} - 6t + 18 + \dfrac{4}{\pi} = \dfrac{4}{\pi}$ at $t = 6$ s, which is the value at $t = 4$ s minus 2, the area of the triangle. q remains at this value for $t > 6$ s.

P1.2.3 Electrons are emitted from a heated metal plate A at a constant rate of 6.25×10^{14} electrons/s, with zero kinetic energy. They are accelerated toward a parallel metal plate B that is separated from A by 5 mm. Plates A and B are connected to an external power supply that maintains B at a constant voltage of $+10\,\text{V}$ with respect to A. (a) How much potential energy does an electron gain or lose in going from A to B? (b) What happens to this potential energy? (c) What is the velocity of the electron when it arrives at B? Assume an electron has a charge of -1.6×10^{-19} C and a mass of 9.1×10^{-31} kg.

Solution: (a) Since electrons move to a more positive voltage, they lose potential energy.

Energy loss per electron $= qV = 1.6\times10^{-19}\times10 = 1.6\times10^{-18}$ J.

(b) It is converted to K.E.

(c) $\frac{1}{2}mv^2 = 1.6\times10^{-18}$ J; $v = \sqrt{\frac{3.2\times10^{-18}}{9.1\times10^{-31}}} = 1.88\times10^{6}$ m/s.

P1.2.4 Consider Problem P1.2.3. (a) What is the total kinetic energy of the electrons that arrive at B during one second? (b) The accelerated electrons are 'collected' at B, where they flow through plate B to the positive plate of the battery. What is the magnitude and direction of the resulting current through the power supply? (c) What happens to the kinetic energy of the electrons once they are collected at B? (d) How much power is expended by the power supply in order to keep the voltage between A and B at $10\,\text{V}$? (e) How is this power related to the kinetic energy given up by the electrons?

Solution: (a) $6.25\times10^{14}\times1.6\times10^{-18} = 10\times10^{-4}$ J $\equiv$ 1 mJ.

(b) Current magnitude is: $6.25\times10^{14}\times1.6\times10^{-19} = 10\times10^{-5}$ A $\equiv$ 100 μA in the direction of a voltage rise through the battery.

(c) The kinetic energy is converted to heat.

(d) $P = V\times I = 10\times10^{-4}$ W $\equiv$ 1 mW.

(e) From (a), K.E. given up per second = 1 mW.

P1.3.5 The voltage drop across a certain device, and the current through it, in the direction of the voltage drop, are shown in Figure P1.3.5. Determine: (a) the charge q through the device at the end of each 1 s interval from $t = 0$ to $t = 6$ s; (b) the instantaneous power p during the aforementioned intervals; and (c) the total energy consumed by the device.

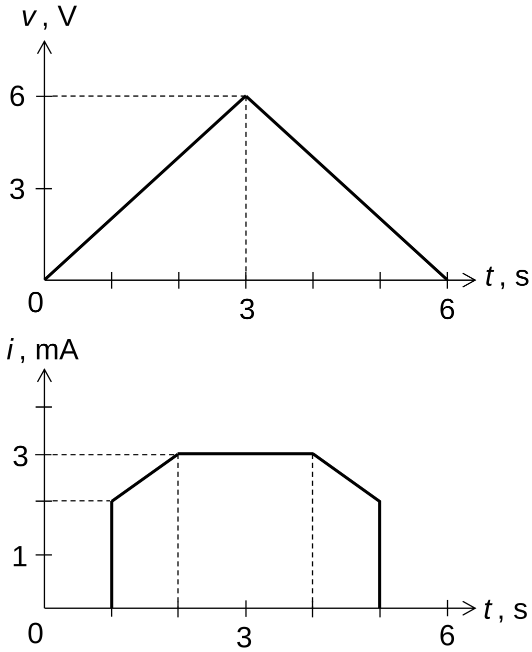

Figure P1.3.5

Solution: (a) <u>0 < t < 1:</u> $q = \int_0^1 idt = \int_0^1 0dt = 0$;

<u>t = 2:</u> $q = \int_1^2 idt = \int_1^2 (1+t)dt =$` $\left[t + \frac{t^2}{2}\right]_1^2$ = 2.5 mC;

<u>t = 3:</u> $q = 2.5 + \int_2^3 idt = 2.5 + \int_2^3 3dt = 5.5$ mC;

<u>t = 4:</u> $q = 5.5 + \int_3^4 idt = 5.5 + \int_3^4 3dt = 8.5$ mC;

<u>t = 5:</u> $q = 8.5 + \int_4^5 idt = 8.5 + \int_4^5 (-t+7)dt = 8.5 + \left[-\frac{t^2}{2} + 7t\right]_4^5$ = 11 mC;

<u>t = 6:</u> $q = 11 + \int_5^6 idt = \int_5^6 0dt = 11$ mC.

(b) <u>$0 \le t \le 1$</u>: $p = 0$;

<u>$1 \le t \le 2$</u>: $p = 2t(t+1) = 2t^2 + 2t$ mW;

<u>$2 \le t \le 3$</u>: $p = 2t \times 3 = 6t$ mW;

<u>$3 \le t \le 4$</u>: $p = (-2t+12) \times 3 = -6t + 36$ mW;

<u>$4 \le t \le 5$</u>: $p = (-2t+12)(-t+7) = 2t^2 - 26t + 84$ mW;

<u>$5 \le t \le 6$</u>: $p = 0$.

(c) $w(t) = \int_0^6 pdt = \int_0^1 0dt + \int_1^2 (2t^2 + 2t)dt + \int_2^3 6tdt + \int_3^4 (-6t + 36)dt +$

$\int_4^5 (2t^2 - 26t + 84)dt + \int_5^6 0dt = \left[\frac{2t^3}{3} + t^2\right]_1^2 + \left[3t^2\right]_2^3 + \left[-3t^2 + 36t\right]_3^4 +$

$\left[\frac{2t^3}{3} - 13t^2 + 84t\right]_4^5$ = 136/3 = 45.3 μJ.

P1.3.7 The voltage drop v V across a certain device, and the current i A through it, in the direction of the voltage drop, are related by:

$$i = 8 - 2v^2, \qquad 0 \le v \le 2\text{ V}$$

$$i = 0, \qquad v \le 0 \text{ and } v \ge 2\text{ V}$$

(a) Determine the power absorbed by the load when v = 1 V, and when v = 2 V;

(b) At what value of v is the instantaneous power a maximum?

(c) If $v(t) = 2e^{-t}$ V, $t \ge 0$ s, what is the total charge that passes through the device from t = 0 to $t = 2$ s?

Solution: (a) $0 \le v \le 2$; $p = vi = v(8 - 2v^2) = -2v^3 + 8v$, and $p = 0$, $v \ge 2$.

At v = 1 V; p = 6 W; at v = 2 V, p = 0.

(b) $\frac{dp}{dv} = -6v^2 + 8$, $0 \le v \le 2$; $p_{max} \Rightarrow \frac{dp}{dv} = 0 \Rightarrow -6v^2 + 8 = 0 \Rightarrow v_{\max} = \frac{2\sqrt{3}}{3}$ V; to show that this is a maximum, $\Rightarrow \frac{d^2p}{dv^2} = -12v$, which is negative at $v_{\max}$.

(c) $v(t) = 2e^{-t} \Rightarrow i = 8 - 8e^{-2t}$; $q = \int_0^2 i dt = \int_0^2 \left(8 - 8e^{-2t}\right)dt = \left[8t + 4e^{-2t}\right]_0^2 = 12.07$ C.

P1.3.8 The current $i(t)$ through a circuit element A and the voltage $v(t)$ across it are as shown in figure P1.3.8.

(a) Determine the total charge passing through A; (b) derive $p(t)$, sketch it, and indicate in which intervals is power absorbed or delivered; (c) determine the total energy absorbed or delivered by A for -1 s $\leq t \leq$ 1 s.

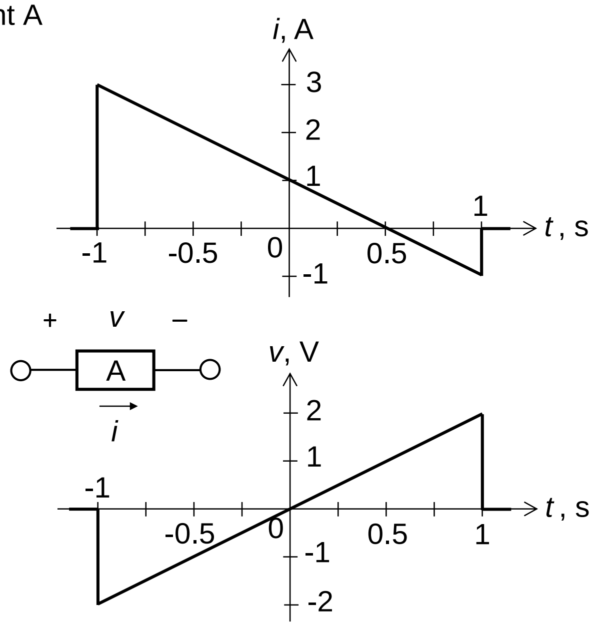

Figure P1.3.8

Solution: (a) From the areas involved, $q = \frac{1}{2} \times 3 \times 1.5 - \frac{1}{2} \times 1 \times 0.5 = 2$ C;

(b) $i(t) = 1 - 2t$, $v(t) = 2t$, $p(t) = 2t - 4t^2$ W;

(c) $w(t) = \int_{-1}^{1} (2t - 4t^2)dt$

$$= \left[t^2 - \frac{4}{3}t^3 \right]_{-1}^{1} = -\frac{8}{3} \text{ J.}$$

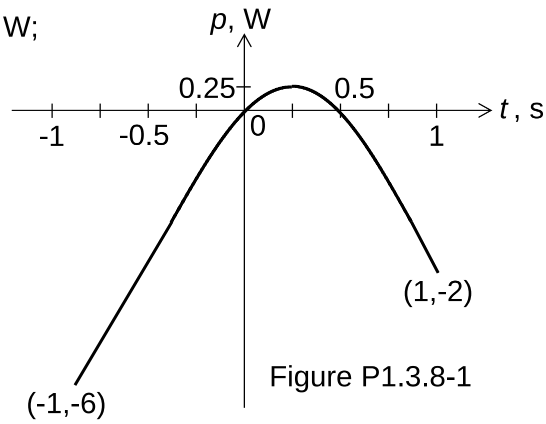

Figure P1.3.8-1